

Noise induced rupture process: Phase boundary and universal scaling of waiting time

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A bundle of fibers has been considered here as a model for composite materials, where breaking of the fibers occur due to a combined influence of applied load (stress) and external noise. Through numerical simulation and a mean-field calculation we show that there exists a robust phase boundary between continuous (no waiting time) and intermittent fracturing regimes. In the intermittent regime, throughout the entire rupture process avalanches of different sizes are produced and there is a waiting time between two consecutive avalanches. The statistics of waiting times follows a Gamma distribution and the avalanche distribution shows power law scaling - similar to what have been observed in case of earthquake events. This suggests that noise induced intermittent rupture process might be the true origin of typical scaling in waiting times and avalanches, independent of the length scale involved.

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Rupture and breakdown [1, 2] are complex processes that occur both in micro and macro scales. Natural rupture phenomena like earthquake, land-slide, mine-collapse, snow-avalanches often appear catastrophic to human society. It is therefore a fundamental challenge to understand the underlying rupture process so that the losses in terms of properties and lives can be minimised by providing early alarms. The same crisis persists in construction engineering and material industry where detail knowledge of the strength of the materials and their failure properties, are essential. But the physical processes which initiate rupture, help its growth and finally results in breakdown, are not completely understood yet.

Fiber bundle model (FBM) has become a useful tool for studying rupture and failure [3] of composite materials under different loading conditions. The simple geometry of the model and clear-cut load-sharing rules allow to achieve analytic solutions [4–6] to an extent that is not possible in any of the fracture models studied so far by the fracture community. FBM was introduced first in connection with textile engineering [7] and recently physicists took interest in it, mainly to explore the critical failure dynamics and avalanche phenomena in this model [8–10]. Not only the classical fracture-failure (stress-induced) in composites, FBM has been used successfully for studying noise-induced (fatigue) failure [11–15] where a fixed load is applied on the system and external noise triggers the failure of elements.

In this Letter we introduce the concept of waiting time for a noise induced intermittent fracturing process in composite materials under fixed external loading. The waiting time is defined as the time (Monte-Carlo steps) between two consecutive avalanches in the avalanche time series for the entire failure process. Through a mean-field calculation we show that in the stress-noise space, there exists a robust phase boundary between continuous (no

waiting time) and intermittent fracturing regimes and that can be verified by numerical simulations. In the intermittent fracturing regime we study the distributions of avalanches and waiting times for different type of fiber strength distributions. Finally to compare our model results with real rupture situation at large scale we analyse earthquake data series (California catalog).

We consider first a bundle of N parallel fibers - and a load ($W = \sigma N$) is applied on the bundle. The fibers have different individual strengths (x) which are drawn from a probability distribution and the bundle has a critical strength σ_c [3], so that without any noise, the bundle does not fail completely for stress $\sigma \leq \sigma_c$, but it fails immediately for $\sigma > \sigma_c$. We now assume that each fiber having strength x_i has a finite probability $P(\sigma, T)$ of failure at any stress σ induced by a nonzero noise T :

$$P(\sigma, T) = \begin{cases} \exp \left[-\frac{1}{T} \left(\frac{x_i}{\sigma} - 1 \right) \right], & 0 \leq \sigma \leq x_i \\ 1, & \sigma > x_i \end{cases} \quad (1)$$

Here $P(\sigma, T)$ increases as T increases and for a fixed value of T and σ_c , as we increase σ , the bundle breaks more rapidly. We simulate this failure phenomenon following Eq. (1) in discrete time t . After each failure (at the fixed stress σ) the total load $N\sigma$ is redistributed among the remaining fibers equally and we check at time $t + 1$, if the present stress $\sigma(t + 1) = W/N(t + 1)$ can induce any further failure following Eq. (1). When the value of σ is considerably large, it so happens that at every time step at least a single fiber breaks until the complete collapse of the bundle. This is a single avalanche and there is no waiting time [15]. But as we decrease the initial value of σ , at a limiting value, in a particular time step t not a single fiber breaks. We consider this as a single waiting time ($t_W = 1$) and the limiting value of σ , at which the waiting time appears for the first time is denoted by σ_0 . This is the onset of intermittent fracturing process.

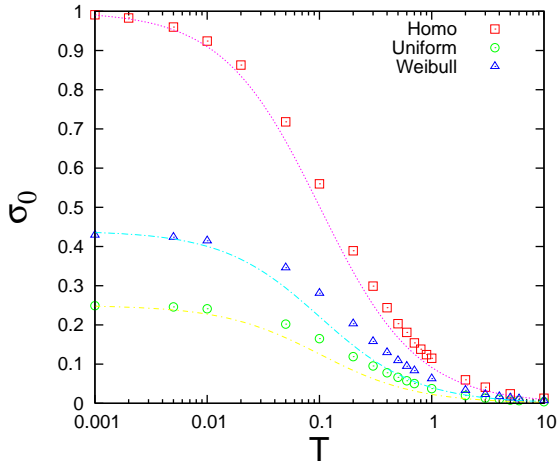


FIG. 1: Phase boundary (σ_0 vs. T plot) for three different type of fiber strength distributions with $N = 20000$. Data points are simulation results and solid lines are analytic estimates (Eqs. 3,4) based on mean-field arguments.

After one waiting time, again another avalanche starts and eventually all the fiber break after such finite number of avalanches. The number of fibers broken during a single avalanche is counted as the avalanche size (m). It is obvious that as we increase the value of T , the value of σ_0 decreases. When the noise is large, the applied load has to be smaller for the emergence of a waiting time. Thus stress (σ) and noise (T) values determine whether the system is in continuous rupture phase or in the intermittent rupture phase.

To determine the phase boundary we can give a mean-field argument that at $\sigma = \sigma_0$, at least one fiber must break to trigger the continuous fracturing process. After this single failure the load has to be redistributed on the intact fibers and the effective stress must be more than σ_0 - which in turn enhances failure probability for all the intact fibers. Therefore in case of homogeneous bundle where all the fibers have identical strength $x_i = 1$ (therefore $\sigma_c = 1$), at the phase boundary $NP(\sigma_0, T) \geq 1$ giving

$$N \exp \left[-\frac{1}{T} \left(\frac{1}{\sigma_0} - 1 \right) \right] \geq 1 \quad (2)$$

which gives

$$\sigma_0 \geq \frac{1}{1 - T \log(1/N)}. \quad (3)$$

In absence of noise T , $\sigma_0 = 1 = \sigma_c$, which is consistent with the static FBM results [3]. This analytic estimate coincides with the data obtained from simulation (Fig. 1). It shows a nice phase boundary between the continuous and intermittent fracturing regimes.

For heterogeneous cases where fibers have different strength and the whole bundle has a critical strength σ_c ,

we make the conjecture that

$$\sigma_0 \geq \frac{\sigma_c}{1 - T \log(1/N)}; \quad (4)$$

keeping in mind that in absence of noise T , $\sigma_0 = \sigma_c$. To verify our conjecture we choose heterogeneous bundles of N fibers where strength of the fibers are drawn from a statistical distribution. We have considered two different kinds of fiber strength distributions: (1) uniform distribution of fiber strength having cumulative form $Q(x) = x$ for $0 < x \leq 1$ and (2) Weibull distribution $Q(x) = 1 - \exp(-x^k)$ where k is the Weibull index (we have taken $k = 2.0$ and 5.0). Each fiber has a finite probability $P(\sigma, T)$ of failure at any stress σ induced by a nonzero T as mentioned before. Similar to the homogeneous case, for a particular value of T , below a certain value of σ , the waiting time appears here. One can see that the theoretical estimate of phase boundary agree with the numerical data for the heterogeneous cases (Fig. 1). However this agreement was much better for homogeneous case. This difference can be explained through the amount of randomness involved in the respective systems. In case of homogeneous bundle there is no randomness in the fiber strength - the only randomness is coming from the noise term. Whereas in case of heterogeneous bundles - there are two sources of randomness - in the fiber strengths and in the noise term.

Existence of such a phase boundary has important consequences on fracturing study in material failure and other fracture-breakdown phenomena. In real situations of material/rock fracturing, acoustic emission measurements can show clearly whether an ongoing fracturing process belongs to continuous or intermittent fracturing phase. Acoustic emissions [16] are basically sound waves produced during micro-crack opening within the material body due to external stress and noise factors. Once a system enters into continuous fracturing phase the breakdown must be imminent. Thus the identification of rupture phase can predict the fate of a system correctly.

In the intermittent fracturing phase avalanches of different size are produced separated by waiting times (t_W) of different magnitude. This happens for a stress value σ below σ_0 at a certain noise (T) level. We have studied the waiting time distribution for both homogeneous and heterogeneous bundles with $N = 20000$. Each curve can be fitted with a Gamma distribution

$$D(t_W) \propto \exp(-t_W/a)/t_W^{1-\gamma} \quad (5)$$

where $\gamma = 0.15$ for homogeneous case and $\gamma = 0.26$ for heterogeneous cases (Fig. 2). The value of a is the measure of the extent of the power law regime and it has different value for different type of strength distribution. We have also studied the waiting time distribution for a fixed value of N , but different sets of values of T and σ , all of which shows Gamma distribution of the form

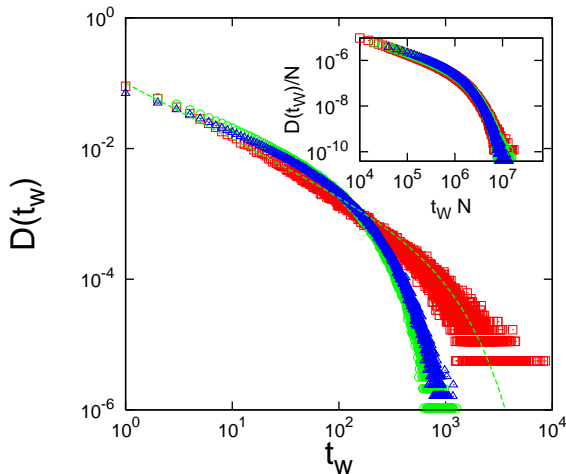


FIG. 2: The simulation results for the waiting time distributions for three different type of fiber strength distributions with $N = 20000$. All the curves can be fitted with the Gamma form $\exp(-t_W/a)/t_W^{1-\gamma}$ (dashed line) where $\gamma = 0.15$ for homogeneous case and $\gamma = 0.26$ for uniform and Weibull distributions. In the inset we show the data collapse of the waiting time distributions with system sizes for uniform distribution.

of Eq. 5. For a fixed value of N and T as σ decreases, the power law region extends longer and thus the value of a increases, but the exponent of power law decay remains same. Again for a certain value of N and σ as T decreases, the value of a increases without any change in the power law exponent. These results imply that the power law exponent remains unchanged with variation of σ , T and N .

In the waiting time distributions, power law part dominates for small t_W values and exponential law dominates for bigger t_W values. The noise-induced rupture process, modeled here, has two basic ingredients, external stress σ and noise T . The noise term triggers initial rupture which induces one or more load-redistribution cycles that finally enhances the effective stress level on the system. Therefore the initial phase of the rupture process is dominated by noise term and as the rupture process goes on stress factor becomes more dominating. At the final stage the stress redistribution mechanism drives the system toward complete collapse through a big avalanche. The inherent global load sharing nature is responsible for the power law part of the Gamma distribution - as power law usually comes from a long range cooperative mechanism [1, 2, 8]. The exponential part of the Gamma distribution is contributed by the noise induced failure factor $P(\sigma, T)$. For large t_W values one can eventually treat the

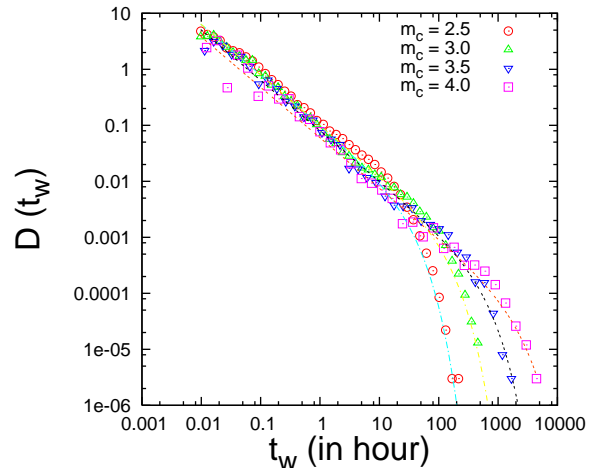


FIG. 3: Gamma-fitting (dotted lines) to the waiting time distributions in California catalog (1984-2002).

failures to be independent. If P indicates the noise induced failure probability within t_W then the probability $D(t_W) = A(1 - P)^{t_W N} \sim \exp(-Pt_W N)$, where A is a constant. The normalization of $D(t_W)$ requires $A \sim N$. Though for smaller values of t_W , one can not ignore the correlations between successive failures (responsible for the power law part in $D(t_W)$), the exponential scaling behavior for $D(t_W)$ can be easily obtained from the above. As shown in the inset of Fig. 2, the plot of $D(t_W)/N$ against $t_W N$ gives good data collapse for different N values. Such a data collapse indicates the robustness of the Gamma function form.

Our model for noise induced rupture process is not limited to any particular system, rather it is a general approach and can model more complex situations like rupture driven earthquakes. There are evidences of stress redistribution and stress-localisation around fracture/fault lines in a active seismic-zone and several factors that can help rupture evolution are friction, plasticity, fluid migration, spatial heterogeneities, chemical reactions etc. In our model such stress redistribution/localisation can be taken into account through a proper load sharing scheme and noise term (T) can represent the combined effect of other factors. To compare the waiting time results of our model system with real data, we have analysed California earthquake catalog from 1984 to 2002 [17]. We are particularly interested in the statistics of waiting times [18–20] between earthquake events. First, we set a cutoff (m_c) in the earthquake magnitude - so that all earthquake events above this cutoff magnitude will be considered for the analysis. The distribution of waiting times shows similar variation for different cutoff values. We get excellent fitting to data points for all the data sets with a Gamma distribution [18]:

$$D(t_W) \propto \exp(-t_W/a)/t_W^{(1-\gamma)}; \quad (6)$$

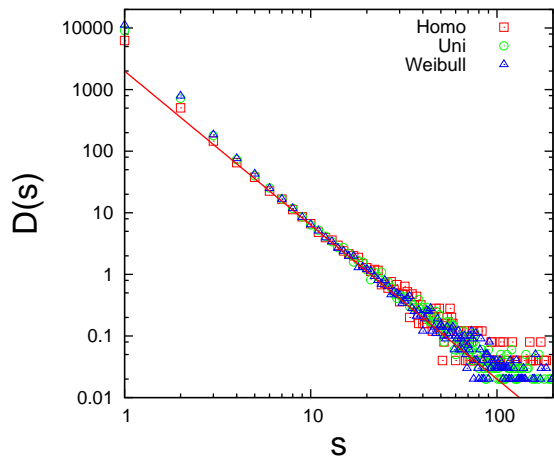


FIG. 4: Numerical data for avalanche size distributions for three different type of fiber threshold distributions with $N = 20000$. The straight line has a slope -2.5 .

with same γ ($\simeq 0.1$) and different a values for different cutoff levels: $a = 30, 120, 500, 2000$ respectively for $m_c = 2.5, 3.0, 3.5, 4.0$ (see Fig. 3).

In general, avalanches or bursts bear important information of the dynamics of intermittent processes. In our model the noise T triggers a rupture process which continues through load (or stress) redistribution mechanism. The avalanche size distributions follow an universal power law ($D(s) \sim s^{-\xi}$) scaling with exponent $\xi = 2.5$. This result (Fig.4) demands that such intermittent rupture process belong to the quasi-static fracturing class, where the universality of the exponent value has already been established [8].

Identification of phase boundary is crucial for any dynamical system because a system usually changes its behavior as it moves from one phase to another. As we can see in our model, there is no waiting time above the phase boundary (continuous rupture phase) and waiting time appears below the phase boundary (intermittent phase). One can also estimate the failure time of the system exactly [15] in the continuous rupture phase.

The concept of waiting time in such a noise-induced rupture process is a new and useful concept which allows us to study the avalanche time series with the spirit and tools that have been used in earthquake catalog analysis. The similarities in waiting time statistics and scaling forms suggest that slowly driven (noise induced) fracturing process and earthquake dynamics (stick-slip mechanism) perhaps have some common origin. In case of fracturing in loaded rocks/materials, such study can help to identify reliable precursors which can warn of an imminent breakdown. We notice, in our model system, magnitude of waiting time reduces gradually towards the breakdown point. What is the form of this variation? Does it depend on the applied stress and noise level? Which one is the more sensitive parameter? These questions

must be answered to develop a prediction scheme based on available precursors prior to failure/breakdown.

Finally, if the avalanche and waiting time data for noise-induced fracturing show some similarities with the earthquake time series - then we can analyse the synthetic time series generated from this model more intensively - compared to the earthquake catalog - where data sets are limited and sometimes not large enough to perform a good statistical analysis. The new findings/tools/concepts from such model analysis can be applied in earthquake catalog analysis to explore new areas - which has not been done before.

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